प्रश्न-पत्र कोड
Q.P. Code **65/4/2**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* | *अधिकतम अंक : **80***
*Time allowed : **3** hours* | *Maximum Marks : **80***

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

*(i) इस प्रश्न-पत्र में 38 प्रश्न हैं । **सभी** प्रश्न **अनिवार्य** हैं ।*

*(ii) यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – खण्ड **क, ख, ग, घ** तथा **ङ** ।*

*(iii) **खण्ड-क** में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं ।*

*(iv) **खण्ड-ख** में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं ।*

*(v) **खण्ड-ग** में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं ।*

*(vi) **खण्ड-घ** में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं ।*

*(vii) **खण्ड-ङ** में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं ।*

*(viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड-**ख** के 2 प्रश्नों में, खण्ड-**ग** के 3 प्रश्नों में, खण्ड-**घ** के 3 प्रश्नों में तथा खण्ड-**ङ** के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

*(ix) कैल्कुलेटर का उपयोग **वर्जित** है ।*

खण्ड – क

इस खण्ड में **20** बहुविकल्पी प्रश्न हैं । प्रत्येक प्रश्न का **1** अंक है । **20 × 1 = 20**

1. रेखाएँ $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ तथा $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$, p के जिस मान के लिए परस्पर लंबवत हैं, वह है :

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) 2 (D) 3

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question Paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question Paper is divided into **five** Sections – Section **A, B, C, D** and **E**.*

(iii) *In **Section A** – Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and Questions no. **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C** – Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Questions no. **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E** – Questions no. **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section **B**, **3** questions in Section **C**, **3** questions in Section **D** and **2** questions in Section **E**.*

(ix) *Use of calculators is **not** allowed.*

SECTION – A

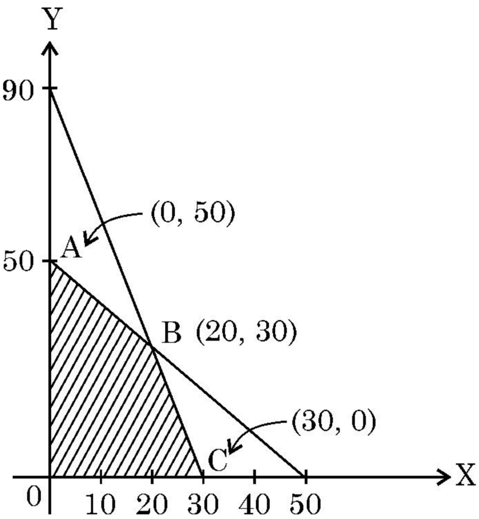
This section consists of **20** multiple choice questions of **1** mark each. **20 × 1 = 20**

1. The lines $\frac{1-x}{2}=\frac{y-1}{3}=\frac{z}{1}$ and $\frac{2x-3}{2p}=\frac{y}{-1}=\frac{z-4}{7}$ are perpendicular to each other for p equal to :

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) 2 (D) 3

2. रैखिक प्रोग्रामन समस्या (LPP) जिसका सुसंगत क्षेत्र दर्शाया गया है, के उद्देश्य फलन $Z = 4x + y$ का अधिकतम मान है :



(A) 50 (B) 110

(C) 120 (D) 170

3. यदि एक यादृच्छिक चर X का प्रायिकता बंटन, निम्न है :

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

जहाँ k एक अज्ञात अचर है ।

तो यादृच्छिक चर X का मान 2 होने की प्रायिकता है

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

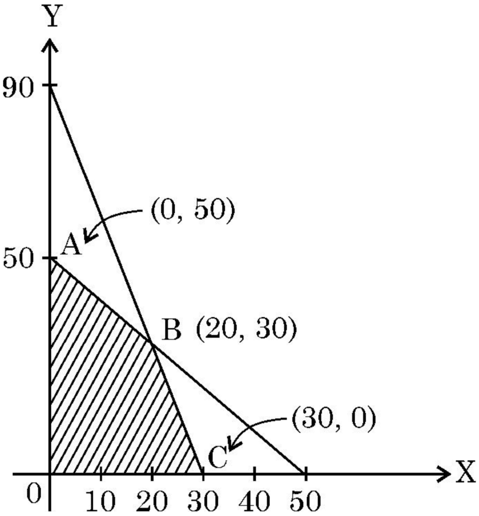
(C) $\frac{4}{5}$ (D) 1

4. यदि $A = [a_{ij}] = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 3 & 2 \\ 5 & 0 & 4 \end{bmatrix}$ है तथा c_{ij} अवयव a_{ij} का सहखण्ड है, तो $a_{21} \cdot c_{11} + a_{22} \cdot c_{12}$ $+ a_{23} \cdot c_{13}$ का मान है :

(A) –57 (B) 0

(C) 9 (D) 57

2. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is :



(A) 50 (B) 110

(C) 120 (D) 170

3. The probability distribution of a random variable X is :

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

where k is some unknown constant.

The probability that the random variable X takes the value 2 is :

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$

(C) $\frac{4}{5}$ (D) 1

4. If $A = [a_{ij}] = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 3 & 2 \\ 5 & 0 & 4 \end{bmatrix}$ and c_{ij} is the cofactor of element a_{ij}, then the value of $a_{21} \cdot c_{11} + a_{22} \cdot c_{12} + a_{23} \cdot c_{13}$ is :

(A) –57 (B) 0

(C) 9 (D) 57

5. यदि $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ है तथा $A^2 - kA - 5I = O$ है, तो k का मान है :

(A) 3
(B) 5
(C) 7
(D) 9

6. यदि $e^{x^2 y} = c$ है, तो $\frac{dy}{dx}$ बराबर है :

(A) $\frac{x e^{x^2 y}}{2y}$
(B) $\frac{-2y}{x}$
(C) $\frac{2y}{x}$
(D) $\frac{x}{2y}$

7. अचर c का वह मान, जिसके लिए $f(x) = \begin{cases} x^2 - c^2, & \text{यदि } x < 4 \\ cx + 20, & \text{यदि } x \geq 4 \end{cases}$ द्वारा परिभाषित फलन f , सभी वास्तविक संख्याओं के लिए संतत है, है :

(A) –2
(B) –1
(C) 0
(D) 2

8. $\int_{-1}^{1} |x| \, dx$ का मान है :

(A) –2
(B) –1
(C) 1
(D) 2

9. अवकल समीकरण $\log\left(\frac{dy}{dx}\right) = 3x + 4y;\ y(0) = 0$ के विशिष्ट हल में उपस्थित स्वेच्छ अचरों की संख्या है :

(A) 2
(B) 1
(C) 0
(D) 3

10. यदि $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ एक अदिश आव्यूह (scalar matrix) है, तो a + 2b + 3c + 4d का मान है

(A) 0
(B) 5
(C) 10
(D) 25

5. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = O$, then the value of k is :

(A) 3 (B) 5

(C) 7 (D) 9

6. If $e^{x^2 y} = c$, then $\frac{dy}{dx}$ is :

(A) $\frac{x e^{x^2 y}}{2y}$ (B) $\frac{-2y}{x}$

(C) $\frac{2y}{x}$ (D) $\frac{x}{2y}$

7. The value of constant c that makes the function f defined by

$$f(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \geq 4 \end{cases}$$

continuous for all real numbers is :

(A) –2 (B) –1

(C) 0 (D) 2

8. The value of $\int_{-1}^{1} |x| \, dx$ is :

(A) –2 (B) –1

(C) 1 (D) 2

9. The number of arbitrary constants in the particular solution of the differential equation

$\log\left(\frac{dy}{dx}\right) = 3x + 4y;\ y(0) = 0$ is/are

(A) 2 (B) 1

(C) 0 (D) 3

10. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of a + 2b + 3c + 4d is :

(A) 0 (B) 5

(C) 10 (D) 25

11. यदि $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$ है, तो $I - A + A^2 - A^3 + \ldots$ है :

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

12. दिया है कि $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$ है, तो आव्यूह A है :

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

13. अवकल समीकरण $(x + 2y^2)\frac{dy}{dx} = y \ (y > 0)$ का समाकलन गुणक है :

(A) $\frac{1}{x}$

(B) x

(C) y

(D) $\frac{1}{y}$

14. रेखा $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$ के लंबवत सदिश है :

(A) $5\hat{i} + \hat{j} + 6k$

(B) $\hat{i} + 3\hat{j} + 5\hat{k}$

(C) $2\hat{i} - 2\hat{j}$

(D) $9\hat{i} - 3\hat{j}$

15. सदिश $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ तथा $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ जिस त्रिभुज की भुजाओं को निरूपित करते हैं, वह है :

(A) एक समबाहु त्रिभुज

(B) एक अधिक-कोण त्रिभुज

(C) एक समद्विबाहु त्रिभुज

(D) एक समकोण त्रिभुज

16. माना $\vec{a}$ एक ऐसा सदिश है जिसके लिए $|\vec{a}| = a$ है, तो

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$$ का मान है :

(A) a^2

(B) $2a^2$

(C) $3a^2$

(D) 0

11. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \ldots$ is :

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

12. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is :

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

13. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y \ (y > 0)$ is :

(A) $\frac{1}{x}$ (B) x

(C) y (D) $\frac{1}{y}$

14. A vector perpendicular to the line $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$ is :

(A) $5\hat{i} + \hat{j} + 6k$ (B) $\hat{i} + 3\hat{j} + 5\hat{k}$

(C) $2\hat{i} - 2\hat{j}$ (D) $9\hat{i} - 3\hat{j}$

15. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of

(A) an equilateral triangle (B) an obtuse-angled triangle

(C) an isosceles triangle (D) a right-angled triangle

16. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of

$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is :

(A) a^2 (B) $2a^2$

(C) $3a^2$ (D) 0

17. यदि $\vec{a}$ तथा $\vec{b}$ दो ऐसे सदिश हैं कि $|\vec{a}| = 1$, $|\vec{b}| = 2$ तथा $\vec{a} \cdot \vec{b} = \sqrt{3}$ है, तो $2\vec{a}$ तथा $-\vec{b}$ के बीच का कोण है :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

18. फलन $f(x) = kx - \sin x$ निरंतर वर्धमान है, यदि

(A) $k > 1$ (B) $k < 1$

(C) $k > -1$ (D) $k < -1$

अभिकथन – तर्क आधारित प्रश्न

प्रश्न संख्या **19** एवं **20** में एक अभिकथन (A) के बाद एक तर्क (R) दिया है । निम्न में से सही उत्तर चुनिए :

(A) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं । तर्क (R) अभिकथन (A) की पूरी व्याख्या करता है ।

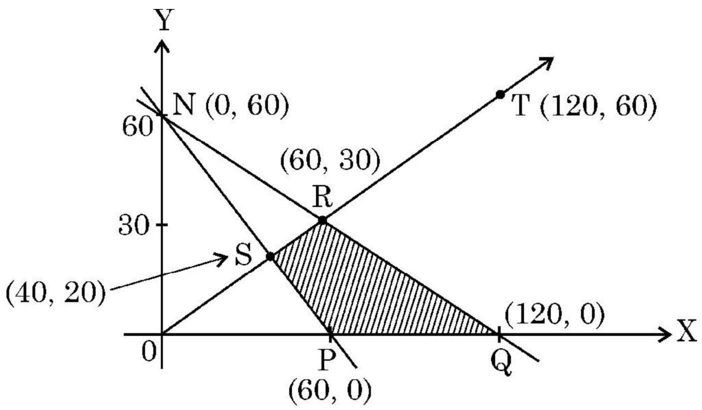
(B) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं । तर्क (R) अभिकथन (A) की पूरी व्याख्या नहीं करता ।

(C) अभिकथन (A) सत्य है, परन्तु तर्क (R) असत्य है ।

(D) अभिकथन (A) असत्य है जबकि तर्क (R) सत्य है ।

19. **अभिकथन (A) :** किसी LPP के लिए परिबद्ध सुसंगत क्षेत्र के कोणीय बिंदु दर्शाए गए हैं ।

$Z = x + 2y$ का अधिकतम मान अनन्त बिंदुओं पर हैं ।



तर्क (R) : एक LPP जिसका सुसंगत क्षेत्र परिबद्ध हो, का इष्टतम हल कोणीय बिंदु पर होता है ।

17. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

18. The function $f(x) = kx - \sin x$ is strictly increasing for

(A) $k > 1$ (B) $k < 1$

(C) $k > -1$ (D) $k < -1$

ASSERTION-REASON BASED QUESTIONS

Questions No. **19** & **20**, are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the codes (A), (B), (C) and (D) as given below :

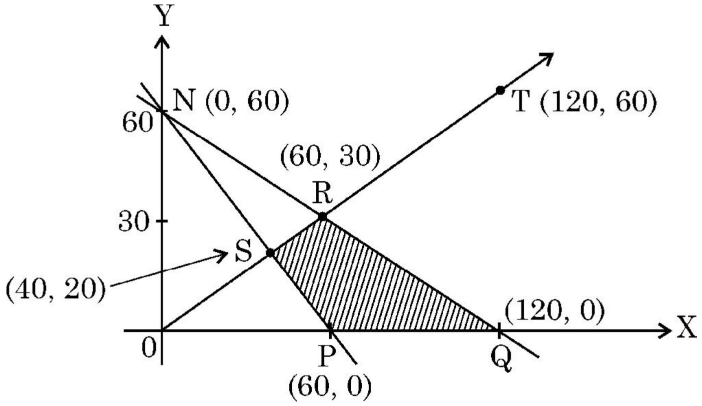
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.



Reason (R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.

20. **अभिकथन (A) :** संबंध $R = \{(x, y) : (x + y)$ एक अभाज्य संख्या है तथा $x, y \in N\}$ एक स्वतुल्य संबंध नहीं है ।

तर्क (R) : सभी प्राकृत संख्याओं n के लिए, 2n एक भाज्य संख्या है ।

खण्ड – ख

इस खण्ड में **5** अति लघु उत्तर वाले प्रश्न हैं, जिनमें प्रत्येक के **2** अंक हैं ।

21. एक घन का आयतन $6\ cm^3/s$ की दर से बढ़ रहा है । घन का पृष्ठीय क्षेत्रफल किस दर से बढ़ रहा है, जब इसके किनारे की लंबाई 8 cm है ?

22. (a) $\frac{-\pi}{2} < x < \frac{\pi}{2}$ के लिए $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$ को सरलतम रूप में व्यक्त कीजिए ।

अथवा

(b) $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ का मुख्य मान ज्ञात कीजिए ।

23. दर्शाइए कि $f(x) = \frac{4 \sin x}{2 + \cos x} - x$; $\left[0, \frac{\pi}{2}\right]$ में x का एक वर्धमान फलन है ।

24. (a) यदि $y = \cos^3 (\sec^2 2t)$ है, तो $\frac{dy}{dt}$ ज्ञात कीजिए ।

अथवा

(b) यदि $x^y = e^{x-y}$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.

25. मान ज्ञात कीजिए : $\int_{\frac{-1}{2}}^{\frac{1}{2}} \cos x \cdot \log\left(\frac{1+x}{1-x}\right) dx$

20. **Assertion (A) :** The relation R = {(x, y) : (x + y) is a prime number and x, y $\in$ N} is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

SECTION – B

In this section there are **5** very short answer type questions of **2** marks each.

21. The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?

22. (a) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

OR

(b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

23. Show that $f(x) = \frac{4\sin x}{2+\cos x} - x$ is an increasing function of x in $\left[0, \frac{\pi}{2}\right]$.

24. (a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

25. Evaluate : $\int_{\frac{-1}{2}}^{\frac{1}{2}} \cos x \cdot \log\left(\frac{1+x}{1-x}\right) dx$

खण्ड – ग

इस खण्ड में **6** लघु-उत्तर प्रकार के प्रश्न हैं, जिनमें प्रत्येक के **3** अंक हैं ।

26. दिया है कि $x^y + y^x = a^b$ है, जहाँ a तथा b धनात्मक अचर हैं, $\frac{dy}{dx}$ ज्ञात कीजिए ।

27. (a) अवकल समीकरण $\frac{dy}{dx} = y \cot 2x$ का विशिष्ट हल ज्ञात कीजिए, दिया है कि $y\left(\frac{\pi}{4}\right) = 2$ ।

अथवा

(b) अवकल समीकरण $(x\, e^{\frac{y}{x}} + y)\, dx = x\, dy$ का विशिष्ट हल ज्ञात कीजिए, दिया है कि y = 1 है जब x = 1 है ।

28. ज्ञात कीजिए : $\int \frac{2x+3}{x^2(x+3)}\, dx$

29. (a) 52 पत्तों की अच्छी प्रकार से फेंटी गई ताश की गड्डी में से एक पत्ता खो जाता है । शेष पत्तों में से यादृच्छया एक पत्ता निकाला जाता है, जो बादशाह वाला पत्ता पाया जाता है । खो गए पत्ते के बादशाह वाला पत्ता होने की प्रायिकता ज्ञात कीजिए ।

अथवा

(b) एक अभिनत पासे पर समसंख्या आने की प्रायिकता, विषम संख्या के आने की प्रायिकता से दुगुनी है । इस पासे को दो बार उछाला गया । छः आने की संख्या का प्रायिकता बंटन ज्ञात कीजिए । इस बंटन का माध्य भी ज्ञात कीजिए ।

30. निम्न रैखीय प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :

व्यवरोधों $x + 2y \leq 200$

$x + y \leq 150$

$y \leq 75$

$x, y \geq 0$

के अंतर्गत $Z = x + 3y$ का अधिकतमीकरण कीजिए ।

SECTION – C

In this section there are **6** short answer type questions of **3** marks each.

26. Given that $x^y + y^x = a^b$, where a and b are positive constants, find $\frac{dy}{dx}$.

27. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

OR

(b) Find the particular solution of the differential equation

$$(xe^{\frac{y}{x}} + y)\, dx = x\, dy, \text{ given that } y = 1 \text{ when } x = 1.$$

28. Find : $\int \frac{2x+3}{x^2(x+3)}\, dx$

29. (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

OR

(b) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

30. Solve the following L.P.P. graphically :

Maximise $Z = x + 3y$

subject to the constraints :

$x + 2y \leq 200$

$x + y \leq 150$

$y \leq 75$

$x, y \geq 0$

31. (a) मान ज्ञात कीजिए : $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{1+\cos 2x+\sin 2x}$

अथवा

(b) ज्ञात कीजिए : $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1+x^2}}\right] dx$

खण्ड – घ

इस खण्ड में **चार** दीर्घ–उत्तर वाले प्रश्न हैं । प्रत्येक प्रश्न के **5** अंक हैं ।

32. (a) माना $A = R - \{5\}$ तथा $B = R - \{1\}$ है । $f(x) = \frac{x-3}{x-5}$ द्वारा परिभाषित फलन $f : A \to B$ पर विचार कीजिए । दर्शाइए कि f एकैकी व आच्छादक है ।

अथवा

(b) जाँच कीजिए कि क्या सभी वास्तविक संख्याओं के समुच्चय R में परिभाषित संबंध

$S = \{(a, b)$: जहाँ $a - b + \sqrt{2}$ एक अपरिमेय संख्या है $\}$

स्वतुल्य, सममित या संक्रामक है ।

33. (a) रेखा $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ तथा इसके समांतर एक अन्य रेखा जो बिंदु $(4, 0, -5)$ से होकर जाती है, के बीच की दूरी ज्ञात कीजिए ।

अथवा

(b) यदि रेखाएँ $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ तथा $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ परस्पर लंबवत हैं, तो k का मान ज्ञात कीजिए । अत: उपरोक्त दोनों रेखाओं के लंबवत एक रेखा का सदिश समीकरण लिखिए, जो बिंदु $(3, -4, 7)$ से होकर जाती है ।

34. आव्यूहों के गुणनफल $\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{pmatrix}\begin{pmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{pmatrix}$ का प्रयोग करते हुए निम्न समीकरण निकाय को हल कीजिए :

$x + 2y - 3z = 6$

$3x + 2y - 2z = 3$

$2x - y + z = 2$

31. (a) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{1+\cos 2x+\sin 2x}$

OR

(b) Find : $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1+x^2}}\right] dx$

SECTION – D

In the section there are **4** long answer type questions of **5** marks each.

32. (a) Let A = R – {5} and B = R – {1}. Consider the function f : A → B, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

OR

(b) Check whether the relation S in the set of real numbers R defined by S = {(a, b) : where a – b + $\sqrt{2}$ is an irrational number} is reflexive, symmetric or transitive.

33. (a) Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5).

OR

(b) If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7).

34. Use the product of matrices $\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{pmatrix}\begin{pmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{pmatrix}$ to solve the following system of equations :

$x + 2y - 3z = 6$

$3x + 2y - 2z = 3$

$2x - y + z = 2$

35. (a) वक्र $y = x|x|$ का आलेख खींचिए । अत: इस वक्र, X-अक्ष तथा कोटियों $x = -2$ तथा $x = 2$ के बीच घिरे क्षेत्र का क्षेत्रफल समाकलन से ज्ञात कीजिए ।

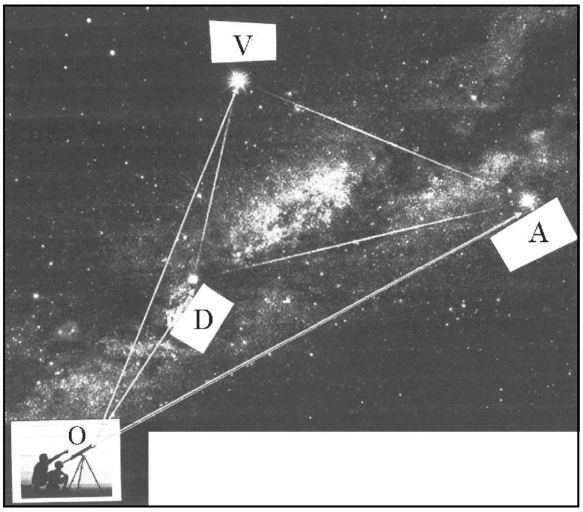
अथवा

(b) समाकलन के प्रयोग से दीर्घवृत्त $9x^2 + 25y^2 = 225$, रेखाओं $x = -2$ तथा $x = 2$ और X-अक्ष के बीच घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

खण्ड – ङ

इस खण्ड में **3** प्रकरण आधारित प्रश्न हैं । प्रत्येक प्रश्न के **4** अंक हैं ।

36. एक खगोलीय केंद्र में एक प्रशिक्षक एक विशेष तारामंडल में सबसे चमकीले तीन सितारों को दर्शाता है । मान लें कि दूरबीन O (0, 0, 0) पर स्थित है तथा तीन सितारों की स्थितियाँ D, A तथा V पर इस प्रकार हैं कि उनके स्थिति-सदिश क्रमश: $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$ तथा $-3\hat{i} + 7\hat{j} + 11\hat{k}$ हैं ।



उपरोक्त के आधार पर निम्न के उत्तर दीजिए :

(i) सितारा V, सितारे A से कितनी दूरी पर है ? **1**

(ii) $\overrightarrow{DA}$ की दिशा में एक एकक-सदिश ज्ञात कीजिए । **1**

(iii) $\angle VDA$ का माप ज्ञात कीजिए । **2**

अथवा

(iii) सदिश $\overrightarrow{DV}$ का सदिश $\overrightarrow{DA}$ पर प्रक्षेप कितना है ? **2**

35. (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

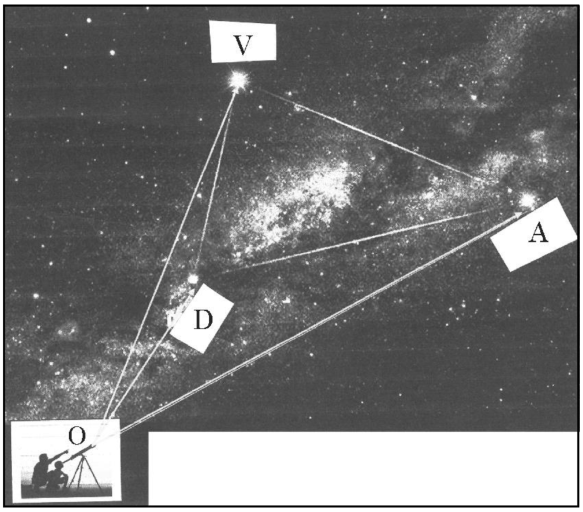
OR

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.

SECTION – E

In this section, there are **3** case study based questions of **4** marks each.

36. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions :

(i) How far is the star V from star A ? **1**

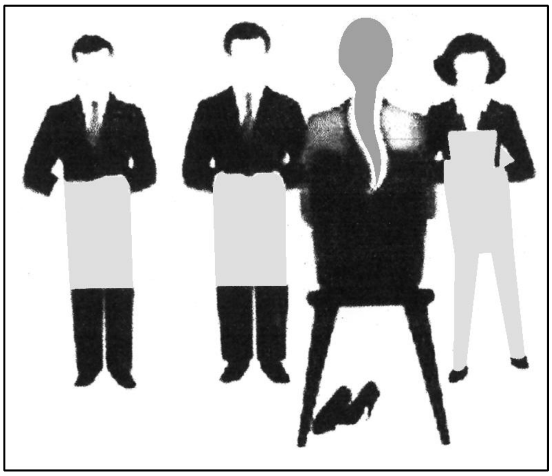
(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. **1**

(iii) Find the measure of $\angle VDA$. **2**

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **2**

37. रोहित, जसप्रीत और आलिया एक ही पद की तीन रिक्तियों के लिए साक्षात्कार के लिए उपस्थित हुए। रोहित के चुने जाने की प्रायिकता $\frac{1}{5}$ है, जसप्रीत के चुने जाने की प्रायिकता $\frac{1}{3}$ तथा आलिया के चुने जाने की प्रायिकता $\frac{1}{4}$ है। चयन की घटना एक दूसरे से स्वतंत्र है।



उपरोक्त जानकारी के आधार पर निम्न प्रश्नों के उत्तर दें :

(i) इनमें से कम से कम एक के चुने जाने की प्रायिकता क्या है ? **1**

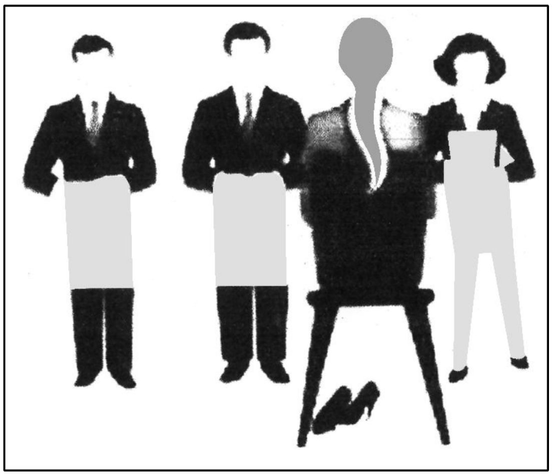
(ii) $P(G \mid \overline{H})$ ज्ञात कीजिए जहाँ G, जसप्रीत के चुने जाने को दर्शाती है तथा $\overline{H}$ रोहित के न चुने जाने को दर्शाती है। **1**

(iii) उनमें से केवल एक के चुने जाने की प्रायिकता ज्ञात कीजिए। **2**

अथवा

(iii) उनमें से कोई दो के चुने जाने की प्रायिकता ज्ञात कीजिए। **2**

37. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions :

(i) What is the probability that at least one of them is selected ? **1**

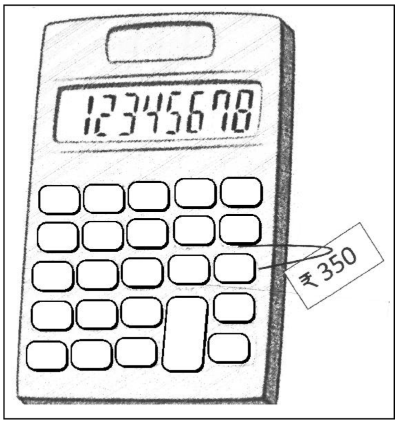
(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. **1**

(iii) Find the probability that exactly one of them is selected. **2**

OR

(iii) Find the probability that exactly two of them are selected. **2**

38. एक स्टोर, कैल्कुलेटर ₹ 350 प्रति कैल्कुलेटर के भाव से बेच रहा है । मार्केट के एक सर्वे के अनुसार मूल्य (p) के घटाने पर बिकने वाले कैल्कुलेटरों की संख्या (x) बढ़ जाती है । मूल्य और बिकने वाली संख्या का संबंध, अर्थात् माँग फलन $p = 450 - \frac{1}{2}x$ द्वारा प्रदत्त है

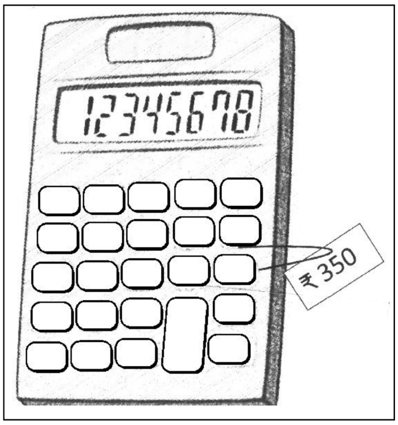


उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

(i) अधिकतम आय $R(x) = xp(x)$ प्राप्त करने के लिए कितनी इकाई (x) बेचने होंगे ? अपने उत्तर का सत्यापन कीजिए ।

(ii) अधिकतम आय के लिए एक कैल्कुलेटर के मूल्य को स्टोर को कितना घटाना होगा ?

38. A store has been selling calculators at ₹ 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.



Based on the above information, answer the following questions :

(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result.

(ii) What rebate in price of calculator should the store give to maximise the revenue ?

Marking Scheme

Strictly Confidential

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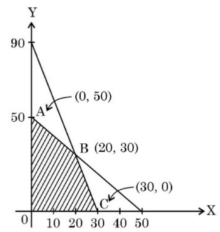
Senior School Certificate Examination, 2024

MATHEMATICS PAPER CODE 65/4/2

General Instructions:

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after delibration and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**

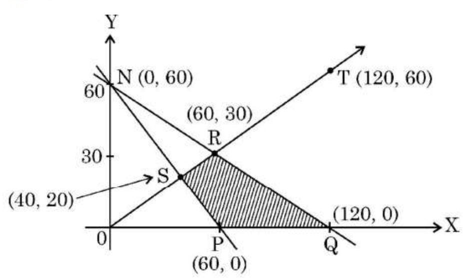
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ____________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

<table>
<tr><th>Q. No.</th><th>EXPECTED OUTCOMES/VALUE POINTS</th><th>Marks</th></tr>
<tr><td></td><td>SECTION A
Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.</td><td></td></tr>
<tr><td>1.</td><td>The lines $\frac{1-x}{2}=\frac{y-1}{3}=\frac{z}{1}$ and $\frac{2x-3}{2p}=\frac{y}{-1}=\frac{z-4}{7}$ are perpendicular to each other for p equal to :
(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$
(C) 2 (D) 3</td><td></td></tr>
<tr><td>Ans:</td><td>(C) 2</td><td>1</td></tr>
<tr><td>2.</td><td>The maximum value of Z = 4x + y for a L.P.P. whose feasible region is given below is


(A) 50 (B) 110 (C) 120 (D) 170</td><td></td></tr>
<tr><td>Ans:</td><td>(C) 120</td><td>1</td></tr>
<tr><td>3.</td><td>The probability distribution of a random variable X is:
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>P(X)</td><td>0.1</td><td>k</td><td>2k</td><td>k</td><td>0.1</td></tr>
</table>
where k is some unknown constant.
The probability that the random variable X takes the value 2 is:
(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{4}{5}$ (D) 1</td><td></td></tr>
<tr><td>Ans:</td><td>(B) $\frac{2}{5}$</td><td>1</td></tr>
<tr><td>4.</td><td>If $A=[a_{ij}]=\begin{bmatrix}2 & -1 & 5\\ 1 & 3 & 2\\ 5 & 0 & 4\end{bmatrix}$ and c_{ij} is the cofactor of element a_{ij}, then the value of $a_{21}.c_{11}+a_{22}.c_{12}+a_{23}.c_{13}$ is :
(A) -57 (B) 0 (C) 9 (D) 57</td><td></td></tr>
<tr><td>Ans:</td><td>(B) 0</td><td>1</td></tr>
</table>

5.	If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = O$, then value of k is : (A) 3 (B) 5 (C) 7 (D) 9	
Ans:	**(B) 5**	**1**
6.	If $e^{x^2 y} = c$, then $\frac{dy}{dx}$ is : $(A)\ \frac{xe^{x^2 y}}{2y}$ $(B)\ \frac{-2y}{x}$ $(C)\ \frac{2y}{x}$ $(D)\ \frac{x}{2y}$	
Ans:	**(B)** $\frac{-2y}{x}$	**1**
7.	The value of constant c that makes the function f defined by $f(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \geq 4 \end{cases}$ continuous for all real numbers is (A) −2 (B) −1 (C) 0 (D) 2	
Ans:	**(A) – 2**	**1**
8.	The value of $\int_{-1}^{1} \lvert x \rvert\, dx$ is: (A) −2 (B) −1 (C) 1 (D) 2	
Ans:	**(C) 1**	**1**
9.	The number of arbitrary constants in the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y;\ y(0) = 0$ is/are (A) 2 (B) 1 (C) 0 (D) 3	
Ans:	**(C) 0**	**1**
10.	If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is (A) 0 (B) 5 (C) 10 (D) 25	
Ans:	**(D) 25**	**1**

11.	If $A=\begin{bmatrix}2 & 1\\ -4 & -2\end{bmatrix}$, then the value of $I-A+A^2-A^3+....$ is $(A)\begin{bmatrix}-1 & -1\\ 4 & 3\end{bmatrix}$ $(B)\begin{bmatrix}3 & 1\\ -4 & -1\end{bmatrix}$ $(C)\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ $(D)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	
Ans:	**(A)** $\begin{bmatrix}-1 & -1\\ 4 & 3\end{bmatrix}$	**1**
12.	Given that $A^{-1}=\frac{1}{7}\begin{bmatrix}2 & 1\\ -3 & 2\end{bmatrix}$, matrix A is: $(A)\ 7\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$ $(B)\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$ $(C)\frac{1}{7}\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$ $(D)\frac{1}{49}\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$	
Ans:	**(B)** $\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$	**1**
13.	The integrating factor of the differential equation $(x+2y^2)\frac{dy}{dx}=y\ \ (y>0)$ is : $(A)\ \frac{1}{x}$ $(B)\ x$ $(C)\ y$ $(D)\ \frac{1}{y}$	
Ans:	**(D)** $\frac{1}{y}$	**1**
14.	A vector perpendicular to the line $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda(3\hat{i}-\hat{j})$ is : (A) $5\hat{i}+\hat{j}+6k$ (B) $\hat{i}+3\hat{j}+5\hat{k}$ (C) $2\hat{i}-2\hat{j}$ (D) $9\hat{i}-3\hat{j}$	
Ans:	**(B)** $\hat{i}+3\hat{j}+5\hat{k}$	**1**
15.	The vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-3\hat{j}-5\hat{k}$ and $\vec{c}=-3\hat{i}+4\hat{j}+4\hat{k}$ represents the sides of (A) an equilaterl triangle (B) an obtuse-angled triangle (C) an isosceles triangle (D) a right-angled triangle	
Ans:	**(D) a right-angled triangle**	**1**
16.	Let $\vec{a}$ be any vector such that $\lvert\vec{a}\rvert=a$. The value of $\lvert\vec{a}\times\hat{i}\rvert^2+\lvert\vec{a}\times\hat{j}\rvert^2+\lvert\vec{a}\times\hat{k}\rvert^2$ is: $(A)\ a^2$ $(B)\ 2a^2$ $(C)\ 3a^2$ $(D)\ 0$	
Ans:	**(B)** $2a^2$	**1**

17.	If $\vec{a}$ and $\vec{b}$ are two vectors such that $\|\vec{a}\|=1, \|\vec{b}\|=2$ and $\vec{a}.\vec{b}=\sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is: $(A)\ \frac{\pi}{6}$ $(B)\ \frac{\pi}{3}$ $(C)\ \frac{5\pi}{6}$ $(D)\ \frac{11\pi}{6}$	
Ans:	**(C)** $\frac{5\pi}{6}$	**1**
18.	The function $f(x) = kx - \sin x$ is strictly increasing for (A) $k > 1$ (B) $k < 1$ (C) $k > -1$ (D) $k < -1$	
Ans:	**(A) k > 1**	**1**
	ASSERTION-REASON BASED QUSTIONS Questions No. 19 & 20, are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct nswer from the codes (A), (B), (C) and (D) as given below: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A). (C) Assertion (A) is true but Reason (R) is false. (D) Assertion (A) is false but Reason (R) is true.	
19.	**Assertion (A) :** The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.  **Reason (R) :** The optimal solution of a LPP having bounded feasible region must occur at corner points.	
Ans:	**(B) Both A and R are true but R is not the correct explanation of A.**	**1**
20.	**Assertion (A) :** The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation. **Reason (R) :** The number '2n' is composite for all natural numbers n.	
Ans:	**(C) Assertion (A) is true, but Reason (R) is false.**	**1**

	SECTION B **In this section there are 5 very short answer type questions of 2 marks each.**	
21.	The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?	
Sol.	Given, $\frac{dV}{dt} = 6\ cm^3/sec$. Since, $V = x^3$ $\frac{dV}{dt} = 3x^2\frac{dx}{dt} \Rightarrow 6 = 3x^2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{2}{x^2} cm/sec$ Now, Surface Area = S = $6x^2 \Rightarrow \frac{dS}{dt} = 12x\frac{dx}{dt} = 3\ cm^2/sec$	**1** **1**
22(a).	Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.	
Sol.	$y = \tan^{-1}\left[\frac{\cos x}{1-\sin x}\right] = \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{(\cos\frac{x}{2} - \sin\frac{x}{2})^2}\right]$ $y = \tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right] = \left(\frac{\pi}{4}+\frac{x}{2}\right)$	**½** **1 ½**
	OR	
22(b).	Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.	
Sol.	$\tan^{-1}(1) + \pi - \cos^{-1}(\frac{1}{2}) - \sin^{-1}(\frac{1}{\sqrt{2}}) = \frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) - \frac{\pi}{4}$ $= \frac{2\pi}{3}$	**1 ½** **½**
23.	Show that $f(x) = \frac{4\sin x}{2+\cos x} - x$ is an increasing function of x in $\left[0, \frac{\pi}{2}\right]$.	
Sol.	$f(x) = \frac{4\sin x}{2+\cos x} - x \Rightarrow f'(x) = \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$ when $x \in [0, \frac{\pi}{2}]$, $\cos x \geq 0 \Rightarrow \frac{\cos x(4-\cos x)}{(2+\cos x)^2} \geq 0$ Thus, f is increasing on $[0, \frac{\pi}{2}]$.	**1** **1**

24 (a).	If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.	
Sol.	$y=\cos^3(\sec^2 2t)$ $\Rightarrow \frac{dy}{dt}=3\cos^2(\sec^2 2t)[-\sin(\sec^2 2t)]\times\frac{d(\sec^2 2t)}{dt}$ $\Rightarrow \frac{dy}{dt}=-3\cos^2(\sec^2 2t).\sin(\sec^2 2t)\times 2\sec 2t.\sec 2t\tan 2t.2$ $\therefore \frac{dy}{dt}=-12\cos^2(\sec^2 2t)\times\sin(\sec^2 2t)\times\sec^2 2t\times\tan 2t.$	½ 1 ½
	OR	
24 (b).	If $x^y = e^{x-y}$, prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$.	
Sol.	$As, x^y=e^{x-y}\Rightarrow \log(x^y)=\log(e^{x-y})$ $\Rightarrow y\log x=(x-y)\Rightarrow y=\frac{x}{1+\log x}$ $Now,$ Differentiating both the sides wrt x $\frac{dy}{dx}=\frac{(\log x+1).1-x(\frac{1}{x})}{(\log x+1)^2}=\frac{\log x}{(1+\log x)^2}$	1 1
25.	Evaluate: $\int_{\frac{-1}{2}}^{\frac{1}{2}}\cos x.\log\left(\frac{1-x}{1+x}\right)dx$	
Sol.	Let $f(x)=\cos x.\log\left(\frac{1-x}{1+x}\right)$ $So, f(-x)=\cos(x).\log\left(\frac{1+x}{1-x}\right)=-f(x)$ [Odd function] Thus, $I=\int_{\frac{-1}{2}}^{\frac{1}{2}}\cos x.\log\left(\frac{1-x}{1+x}\right)dx=0$	1 1
	SECTION C **In this section there are 6 short answer type questions of 3 marks each.**	
26.	Given that $x^y+y^x=a^b$, where a and b are positive constants, find $\frac{dy}{dx}$.	

Sol.	Let u $= x^y \Rightarrow \frac{du}{dx} = x^y(\frac{y}{x} + \log x.\frac{dy}{dx})$, v $=$ y$^x \Rightarrow \frac{dv}{dx} = y^x(\frac{x}{y}\frac{dy}{dx} + \log y)$ Since, u + v $= a^b \Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$ $\therefore \frac{dy}{dx} = \frac{-\left(x^{y-1}y + y^x \log y\right)}{\left(x^y \log x + y^{x-1}x\right)}$	**1 + 1** **1**
27(a).	Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.	
Sol.	$\frac{dy}{dx} = y\cot 2x \Rightarrow \int \frac{dy}{y} = \int \cot 2x dx$ $\Rightarrow \log\|y\| = \frac{1}{2}\log\|\sin 2x\| + \log c$ Thus, $y = c.\sqrt{\sin 2x}$ when $y(\frac{\pi}{4}) = 2$, gives $c = 2$ $\therefore y = 2\sqrt{\sin 2x}$ is the required Particular solution of given D.E.	**1** **1** **½** **½**
	OR	
27(b).	Find the particular solution of the differential equation $(xe^{\frac{y}{x}} + y)\,dx = x\,dy$, given that y = 1 when $x = 1$.	
Sol.	$\frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x} = f(\frac{y}{x})$ so, its a homogeneous differential equation Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ Now, $v + x\frac{dv}{dx} = e^v + v$ $\Rightarrow \int e^{-v}dv = \int \frac{1}{x}dx$ $\Rightarrow -e^{-v} = \log\|x\| + c \Rightarrow -e^{\frac{-y}{x}} = \log\|x\| + c......(1)$ Now, $x = 1,\ y = 1$,gives $c = -e^{-1}$ Thus, $\boxed{\log\|x\| + e^{\frac{-y}{x}} = e^{-1}}$	1 **½** **1** **½**
28.	Find : $\int \frac{2x+3}{x^2(x+3)}\,dx$	

Sol.	$I=\int\frac{2x+3}{x^2(x+3)}dx=\int\frac{x+3}{x^2(x+3)}dx+\int\frac{x}{x^2(x+3)}dx$ $I=\int\frac{1}{x^2}dx+\frac{1}{3}\int\frac{x+3-x}{x(x+3)}dx=\int\frac{1}{x^2}dx+\frac{1}{3}\int\frac{1}{x}dx-\frac{1}{3}\int\frac{1}{x+3}dx$ $I=\frac{-1}{x}+\frac{1}{3}\log\lvert x\rvert-\frac{1}{3}\log\lvert x+3\rvert+c$	1 ½ 1 ½
29(a).	A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.	
Sol.	Let E_1 be the event of lost card is King, E_2 be the event of lost card not a King and A be the event of drawing a King from remaining 51 cards. so, $P(E_1)=\frac{1}{13}, P(E_2)=\frac{12}{13}, P(A\mid E_1)=\frac{3}{51}, P(A\mid E_2)=\frac{4}{51}$ Now, Required probability is $P(E_1\mid A)$, $P(E_1\mid A)=\frac{P(A\mid E_1)\times P(E_1)}{P(A\mid E_1)\times P(E_1)+P(A\mid E_2)\times P(E_2)}=\frac{\frac{1}{13}\times\frac{3}{51}}{\frac{1}{13}\times\frac{3}{51}+\frac{12}{13}\times\frac{4}{51}}=\frac{1}{17}$	½ 1 ½ 1
	OR	
29(b).	A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.	
Sol.	Let P(1)=P(3)=P(5) = p, so P(2)=P(4)=P(6) =2 p As, $P(1)+P(2)+P(3)+P(4)+P(5)+P(6)=1\Rightarrow 9p=1\Rightarrow p=\frac{1}{9}$ $P(\text{Getting } 6)=\frac{2}{9}, P(\text{Not getting six})=\frac{7}{9}$ Let X represents the Number of sixes Possible values of X are 0, 1 or 2 Now, $P(X=0)=\frac{7}{9}\times\frac{7}{9}=\frac{49}{81}, P(X=1)=2\times\frac{7}{9}\times\frac{2}{9}=\frac{28}{81}, P(X=2)=\frac{2}{9}\times\frac{2}{9}=\frac{4}{81}$ Required probability distribution of number of sixes is	½ ½

X	0	1	2
$P(X)$	$\frac{49}{81}$	$\frac{28}{81}$	$\frac{4}{81}$

1½

Mean of X $= \sum_{i=1}^{3} X_i P(X_i) = 0 + \frac{28}{81} + \frac{8}{81} = \frac{36}{81} = \frac{4}{9}$

½

30. Solve the following linear programming problem graphically :

Maximise Z = $2x + 3y$

subject to the constraints :

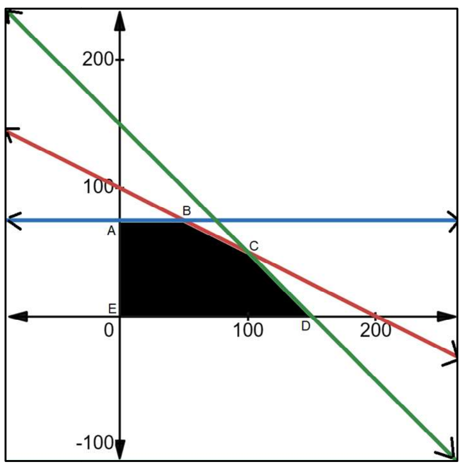
$x + y \leq 6$

$x \geq 2$

$y \leq 3$

$x, y \geq 0$

Sol. On plotting the graph of $x + 2y \leq 200, x + y \leq 150,\ y \leq 75, \& x \geq 0, y \geq 0$ we get the following graph and common shaded region is the region ABCDE.



For correct Graph 1 ½

Now, Corner points of the common shaded region are A(0, 75), $B(50,75), C(100,50), D(150,0)$ & $E(0,0)$.Thus,

Corner points	Value of $Z = x + 3y$
A(0, 75)	225
$B(50,75)$	275
$C(100,50)$	250
$D(150,0)$	150
$E(0,0)$	0

For correct Table 1

So, Maximum Value of Z is 275 at $x = 50$ & $y = 75$.

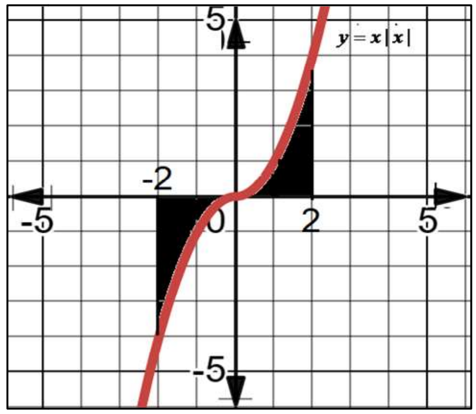
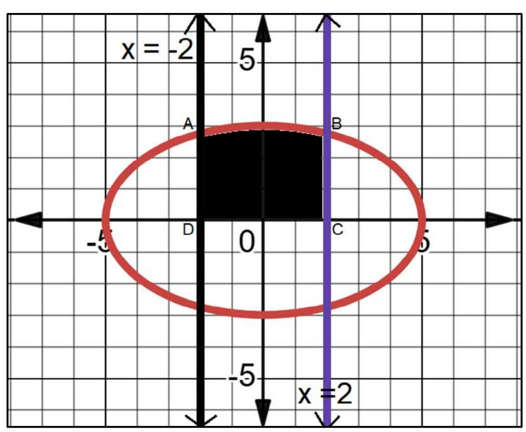
½

31(a).	Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x}{1+\cos 2x+\sin 2x}dx$	
Sol.	$I=\int_0^{\frac{\pi}{4}} \frac{x}{1+\cos 2x+\sin 2x}dx$ ....(1) *On* applying $\int_0^a f(x)dx=\int_0^a f(a-x)dx,$ we get $I=\int_0^{\frac{\pi}{4}} \frac{\frac{\pi}{4}-x}{1+\cos 2x+\sin 2x}dx$...(2) *On* adding Eq. (1) and (2), we get $2I=\frac{\pi}{4}\int_0^{\frac{\pi}{4}} \frac{1}{1+\cos 2x+\sin 2x}dx$ $I=\frac{\pi}{16}\int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x+\sin x\cos x}dx=\frac{\pi}{16}\int_0^{\frac{\pi}{4}} \frac{\sec^2 xdx}{1+\tan x}$ $I=\frac{\pi}{16}(\log\|1+\tan x\|)_0^{\frac{\pi}{4}}$ $I=\frac{\pi}{16}\log 2$	½ ½ 1 ½ ½
	OR	
31(b).	Find : $\int e^x\left[\frac{1}{(1+x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1+x^2}}\right]dx$	
Sol.	$I=\int e^x(\frac{x}{\sqrt{1+x^2}}+\frac{1}{(1+x^2)^{\frac{3}{2}}})dx$ Let $f(x)=\frac{x}{\sqrt{1+x^2}}$, $f'(x)=\frac{\sqrt{1+x^2}-x\frac{x}{\sqrt{1+x^2}}}{1+x^2}=\frac{1+x^2-x^2}{(1+x^2)\sqrt{1+x^2}}=\frac{1}{(1+x^2)^{\frac{3}{2}}}$ On applying $\int e^x(f(x)+f'(x))dx=e^x f(x)+c,\ \ I=e^x\frac{x}{\sqrt{1+x^2}}+c$	½ 1 ½ 1
	SECTION D **In this section there are 4 long answer type questions of 5 marks each.**	
32(a).	Let A = R – {5} and B = R – {1}. Consider the function f : A → B, defined by $f(x)=\frac{x-3}{x-5}$. Show that f is one-one and onto.	

Sol.	Let $f(x_1)=f(x_2)$, for some $x_1, x_2 \in A$ $\Rightarrow \frac{x_1-3}{x_1-5}=\frac{x_2-3}{x_2-5}$ $\Rightarrow (x_1-3)(x_2-5)=(x_2-3)(x_1-5)$ $\Rightarrow x_1=x_2$, So f *is one-one Function.* Let $y=f(x)=\frac{x-3}{x-5} \Rightarrow y(x-5)=x-3$ $\Rightarrow yx-5y=x-3$ $\Rightarrow x=\frac{5y-3}{y-1}$, We observe that x is defined for all values of y except $y=1$, So, Range $=R-\{1\}$ and Co-domain is Given $R-\{1\}$ [As, $f: A \to B$] Since, Range $=$ Co-domain, f is onto Function. Thus, f is one-one & onto function.	**2 ½** **2 ½**
	OR	
32(b).	Check whether the relation S in the set of real numbers R defined by S = {(a, b) : where a – b + $\sqrt{2}$ is an irrational number} is reflexive, symmetric or transitive.	
Sol.	Reflexive: For $a \in S$ $\Rightarrow a-a+\sqrt{2}$ is irrational number $\Rightarrow \sqrt{2}$ is irrational number $\Rightarrow (a,a)\in S$ Thus, S is Reflexive Relation. Symmetric: Let $(a,b)\in S \Rightarrow a-b+\sqrt{2}$ is irrational number but $b-a+\sqrt{2}$ may not be irrational number For example, $(\sqrt{2},1)\in S \Rightarrow \sqrt{2}-1+\sqrt{2}=2\sqrt{2}-1$ is irrational number $(1,\sqrt{2})\notin S$ as $1-\sqrt{2}+\sqrt{2}=1$ is not irrational number $\therefore (b,a)\notin S$, So S is NOT Symmetric Relation. Transitive: *Let* $(a,b)\in S \Rightarrow a-b+\sqrt{2}$ is irrational number $\&(b,c)\in S \Rightarrow b-c+\sqrt{2}$ is irrational number but $a-c+\sqrt{2}$ may not be irrational number *For* example, $(1,\sqrt{3})\in S \Rightarrow 1-\sqrt{3}+\sqrt{2}$ is irrational number $(\sqrt{3},\sqrt{2})\in S \Rightarrow \sqrt{3}-\sqrt{2}+\sqrt{2}=\sqrt{3}$ is irrational number But $(1,\sqrt{2})\notin S$ as $1-\sqrt{2}+\sqrt{2}=1$ is not irrational number $\therefore (a,c)\notin S$, *So S is* NOT Transitive Relation. Thus, S is Reflexive But Neither Symmetric nor Transitive Relation.	**1 ½** **1 ½** **2**

33(a).	Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5).	
Sol.	Equation of the given line in standard form is $L_1: \frac{x}{2}=\frac{y-3}{2}=\frac{z-1}{1}$ Equation of the line parallel to L_1 & passing through $(4,\ 0,-5)$ is $L_2: \frac{x-4}{2}=\frac{y}{2}=\frac{z+5}{1}$ Vector Equation of Lines are $L_1: \vec{r}=(0\hat{i}+3\hat{j}+\hat{k})+\lambda(2\hat{i}+2\hat{j}+\hat{k})$ $L_2: \vec{r}=(4\hat{i}+0\hat{j}-5\hat{k})+\mu(2\hat{i}+2\hat{j}+\hat{k})$ Now, $\overrightarrow{a_2}-\overrightarrow{a_1}=(4\hat{i}+0\hat{j}-5\hat{k})-(0\hat{i}+3\hat{j}+\hat{k})=(4\hat{i}-3\hat{j}-6\hat{k})$ $\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$ $(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -3 & -6\\ 2 & 2 & 1\end{vmatrix}=9\hat{i}-16\hat{j}+14\hat{k}$ $\lvert\vec{b}\rvert=\sqrt{4+4+1}=3$ Thus, Distance between the lines is S.D. $=\frac{\lvert(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}\rvert}{\lvert\vec{b}\rvert}=\frac{\sqrt{81+256+196}}{3}=\frac{\sqrt{533}}{3}$ units	½ 1 ½ 1 ½ 1 ½
	OR	
33(b).	If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7).	
	$L_1: \frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2} \Rightarrow$ direction ratio's of $L_1 = \langle -3, 2k, 2\rangle$ $L_2: \frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7} \Rightarrow$ direction ratio's of $L_2 = \langle 3k, 1, -7\rangle$ Since $L_1 \perp L_2$, $-9k+2k-14=0 \Rightarrow k=-2$	½ ½ 1

	Thus, d.r.'s of $L_1 = <-3,-4,2>$, d.r.'s of $L_2 = <-6,1,-7>$ Now the vector perpendicular to both L_1 & L_2 is given by $\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 2 \\ -6 & 1 & -7 \end{vmatrix} = 26\hat{i} - 33\hat{j} - 27\hat{k}$ Thus, Equation of the required line is $\vec{r} = (3\hat{i} - 4\hat{j} + 7\hat{k}) + \lambda(26\hat{i} - 33\hat{j} - 27\hat{k})$	2 1
34.	Use the product of matrices $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix}$ to solve the following system of equations : $x + 2y - 3z = 6$ $3x + 2y - 2z = 3$ $2x - y + z = 2$	
Sol.	$AB = \begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix} = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix} = 7I$ Thus, $A^{-1} = \frac{1}{7}B = \frac{1}{7}\begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix}$ *so*, Given equation can be written into a matrix equation as $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \Rightarrow X = A^{-1}.C$ A X = C $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 7 \\ -35 \\ -35 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ -5 \end{pmatrix}$ $\therefore x = 1, y = -5, z = -5$	2 1 ½ 1 ½
35(a).	Sketch the graph of $y = x\|x\|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.	
Sol.		

	 As, $y = x\|x\| = \begin{cases} -x^2, x \le 0 \\ x^2, x \ge 0 \end{cases}$ Area of the shaded region $= \int_{-2}^{2} y\ dx = 2\int_{0}^{2} y\ dx = 2\int_{0}^{2} x^2\ dx$ $= 2\left(\frac{x^3}{3}\right)_0^2$ $= 2\left(\frac{8}{3}\right) = \frac{16}{3}$	**For correct figure 1 ½** **1 ½** **1** **1**
	OR	
35(b).	Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.	
Sol.	 	**For correct figure 1 ½**

As, $9x^2 + 25y^2 = 225 \Rightarrow \boxed{y = \pm\frac{3}{5}\sqrt{5^2 - x^2}}$

Required Area $= \int_{-2}^{2} \frac{3}{5}\sqrt{5^2 - x^2}\, dx = \frac{6}{5}\int_{0}^{2} \sqrt{5^2 - x^2}\, dx$ **1 ½**

$$= \frac{6}{5}\left(\frac{x\sqrt{5^2 - x^2}}{2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right)_0^2$$ **1**

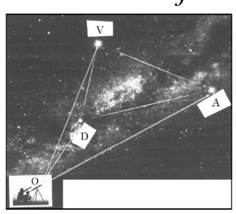
$$= \frac{6}{5}\left(\frac{2\sqrt{21}}{2} + \frac{25}{2}\sin^{-1}\left(\frac{2}{5}\right)\right)$$

$$= \left(\frac{6\sqrt{21}}{5} + 15\sin^{-1}\left(\frac{2}{5}\right)\right)$$ **1**

SECTION E

In this section there are 3 case-study based questions of 4 marks each.

36. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0,0,0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}, 7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions:

(i) How far is the star V from star A? **1**

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. **1**

(iii) Find the measure of $\angle$VDA. **2**

OR

What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **2**

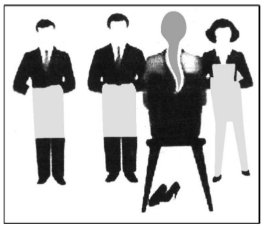
Sol.

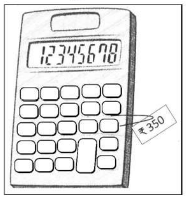
(i) $\overrightarrow{AV}$ = Position Vector of V − Position Vector of A $= -10\hat{i} + 2\hat{j} + 3\hat{k}$ **½**

Thus, $|\overrightarrow{AV}| = \sqrt{100 + 4 + 9} = \sqrt{113}$ units **½**

(ii) $\overrightarrow{DA}$ = Position Vector of A − Position Vector of D $= 5\hat{i} + 2\hat{j} + 4\hat{k}$ **½**

Unit vector in the direction of $\overrightarrow{DA} = \frac{5\hat{i} + 2\hat{j} + 4\hat{k}}{3\sqrt{5}}$ **½**

	(iii) $\overrightarrow{DV} = -5\hat{i} + 4\hat{j} + 7\hat{k}$ $\angle VDA = \cos^{-1}\left(\dfrac{\overrightarrow{DV}.\overrightarrow{DA}}{\lvert\overrightarrow{DV}\rvert\lvert\overrightarrow{DA}\rvert}\right) = \cos^{-1}\left(\dfrac{11\sqrt{2}}{90}\right)$ *OR* (iii) $\overrightarrow{DV} = -5\hat{i} + 4\hat{j} + 7\hat{k}$ Projection of $\overrightarrow{DV}$ on $\overrightarrow{DA} = \left(\dfrac{\overrightarrow{DV}.\overrightarrow{DA}}{\lvert\overrightarrow{DA}\rvert}\right) = \dfrac{11\sqrt{5}}{15}$	½ 1 ½ ½ 1 ½
37.	Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.  Based on the above information, answer the following questions: (i) What is the probability that at least one of them is selected ? (ii) Find P(G|$\overline{\text{H}}$) where G is the event of Jaspreet's selection and $\overline{\text{H}}$ denotes the event that Rohit is not selected. (iii) Find the probability that exactly one of them is selected. OR (iii) Find the probability that exactly two of them are selected.	**1** **1** **2** **2**
Sol.	Given P(Rohit) = $\frac{1}{5}$, P(Jaspreet) = $\frac{1}{3}$, P(Alia) = $\frac{1}{4}$ (*i*) P(atleast one of them is selected) = 1 − P(no one is selected) $= 1 - \left(\frac{4}{5} \times \frac{2}{3} \times \frac{3}{4}\right) = \frac{3}{5}$ (*ii*) P(G|$\overline{\text{H}}$) = $\dfrac{P(G \cap \overline{H})}{P(\overline{H})} = \dfrac{1}{3}$ (*iii*) P(exactly one of them selected) $= P(R) \times P(\overline{J}) \times P(\overline{A}) + P(\overline{R}) \times P(J) \times P(\overline{A}) + P(\overline{R}) \times P(\overline{J}) \times P(A)$ $= \dfrac{6+12+8}{60} = \dfrac{13}{30}$	**1** **1** **1** **1**

	OR (*iii*) P(exactly two of them selected) $= P(R)\times P(J)\times P(\overline{A}) + P(R)\times P(\overline{J})\times P(A) + P(\overline{R})\times P(J)\times P(A)$ $= \frac{3+2+4}{60} = \frac{3}{20}$	 **1** **1**
38.	A store has been selling calculators at Rs. 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by demand function $p = 450 - \frac{x}{2}$.  Based on the above information, answer the following questions: (i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also verify the result. (ii) What rebate in price of calculator should the store give to maximise the revenue?	
Sol.	(i) Revenue by selling x items $= R(x) = x.p(x) = 450x - \frac{x^2}{2}$ $\frac{dR}{dx} = 450 - x$ For Maxima or Minima, $\frac{dR}{dx} = 0 \Rightarrow x = 450$ $\frac{d^2R}{dx^2} = -1 < 0$ (Revenue is maximum when x = 450 units are sold) (*ii*) At $x = 450$, $p = 450 - \frac{450}{2} = 225$ So, Rebate $= 350 - 225 = \text{Rs.}125$ per calculator	**½** **1** **½** **1** **1**